APRIL/MAY 2023

GEMA14C — GRAPH THEORY

Time : Three hours Maximum : 75 marks

Shanmuga Industries Arts and Science College (Co-Ed.) Library
Tiruvannamalai.
Pin: 606 603.

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Adjacency Matrices.
2. State Cayley's formula.
3. What do you mean by Euler tours?
4. Draw Hamilton Cycle.
5. Write Edge Chromatic Number.
6. State Vizing's Theorem.
7. Give Chromatic polynomial.
8. State Ramsey's Theorem.
9. Write Euler's Formula.
10. Define Dual graphs.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Prove that Every nontrivial loopless connected graph has at least two vertices that are not cut vertices.

Or

(b) Prove that In a tree, any two vertices are connected by a unique path.

12. (a) Prove that If G is a nonhamiltonian simple graph with $\upsilon \geq 3$, then G is degree-majorised by some $C_{m,\upsilon}$.

Or

(b) Prove that If G is a simple graph with $\upsilon \geq 3$ and $\delta \geq \upsilon/2$, then G is hamiltonian.

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13. (a) If G is bipartite, prove that $\chi' = \Delta$

Or

(b) Let G be a connected graph that is not an odd cycle. Prove that G has a 2 — edge colouring in which both colours are represented at each vertex of degree at least two.

14. (a) If G is k — critical, prove that $\delta \geq k-1$.

Or

(b) Prove that in a critical graph, no vertex cut is a clique.

15. (a) Show that K_5 is nonplanar.

Or

(b) If G is a plane graph, prove that $\sum_{t \in F} d(F) = 2\varepsilon$

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that An edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

17. Show that C(G) is well defined.

18. If G is simple, prove that either $\chi' = \Delta \, or \, \chi' = \Delta + 1$.

19. If $\delta > 0$, prove that $\alpha' + \beta' = \upsilon$.

20. Show that every planar graph is 5 — vertex — colourable.
